INTERFERENCE

Interference :-
The modification in the distribution of light energy due to superposition of two or more coherent light waves is called "Interference".

Coherent sources :- The light sources having same wave length, same intensity and same phase or a constant phase difference are called the coherent sources. Examples (i) Young double slits. (ii) Llyod's single mirror, source and its virtual images (iii) Biprism two virtual images, etc.

Conditions for sustained interference of light waves.
(i) The two interfering sources must be coherent.
(ii) The interfering waves must have equal amplitudes.
(iii) The two interfering waves must be propagated along the same line.
(iv) The two sources must be narrow.
(v) The separation between the two sources must be very small
(vi) The two interfering sources should emit light of the same frequency or wavelength.

Interference phenomenon, two types.
(i) The wavefront division (ii) The amplitude division.

The interference due to wave front division experiments.
(i) Young's double slit (ii) Llyod's single mirror
(iii) Fresnels Biprism

The interference due to amplitude division experiments.
(i) The Newton's rings (ii) The wedge method (iii) The michelson's interferometer.

Young's double slit experiment :- The experimental arrangement is shown fig ①. Here S is a narrow ~~slit~~ with a source of monochromatic light behind it. S_1 and S_2 are two narrow slits parallel to S. S_1 and S_2 are the coherent sources.

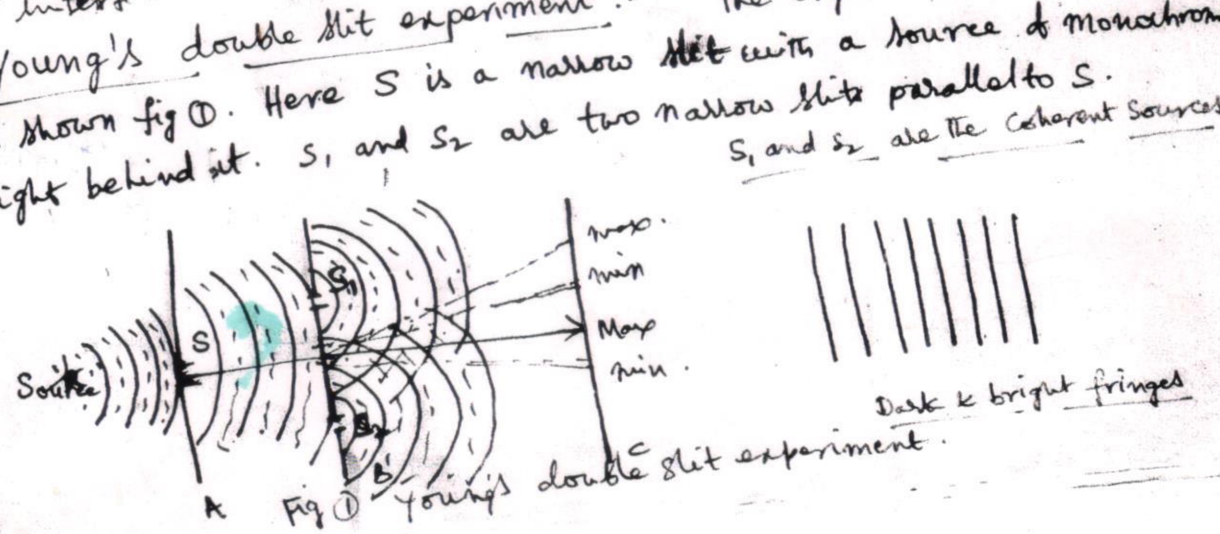


Fig ① Young's double slit experiment.

Fresnel's Biprism :-

The fresnel's Biprism experiment is shown in fig (1). It consists of two small angled prisms ABC and ABD are joined along AB. ∠A. is about 179°. The angles ∠ACB and ∠ADB are about 30′ (α).

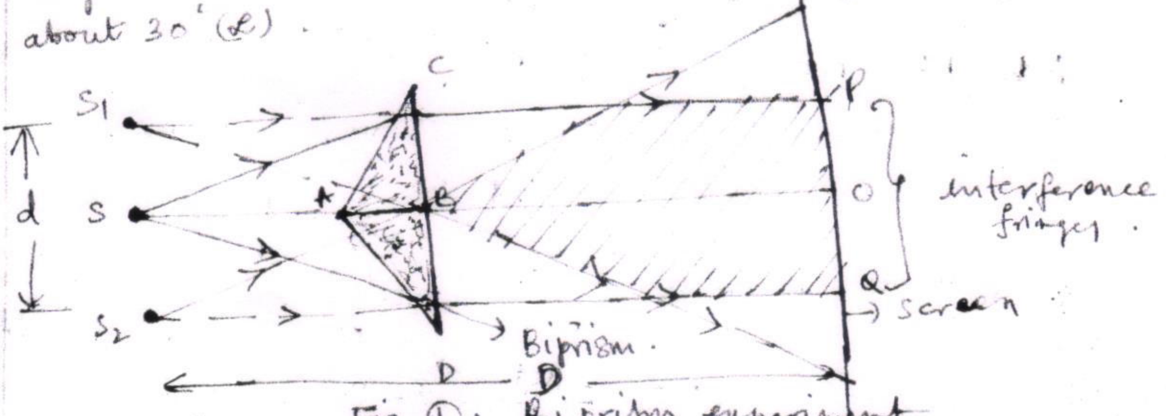


Fig. (1). Biprism experiment

'S' is a narrow slit illuminated by monochromatic light of wavelength λ. The beam of light diverging from S falls symmetrically on the faces AC and AD. These beams deviated towards the base AB, and appear to be diverging from two virtual sources S_1 and S_2. The virtual images S_1 and S_2 act as two coherent sources. In the region PQ of the screen, light reaches from both S_1 and S_2. Hence, interference fringes are obtained in the region PQ. The fringe width $\boxed{\beta = \frac{\lambda D}{d}}$.

In case of Biprism the central maximum is bright fringe.

Colours of **Thin films** :- Thin transparent films like soap film or oil spread over water surface when exposed to an extended source of light produces **colours**. It is due to interference phenomenon.

Consider film of thickness t, refractive index μ. A ray AB incident at an angle 'i', it is partly reflected along BC partly refracted along BD; angle of refraction is 'r'. At 'D' it is again partly reflected along DE inside the medium and partly refracted out of the medium along DK parallel to AB (as the medium above and below is the same).

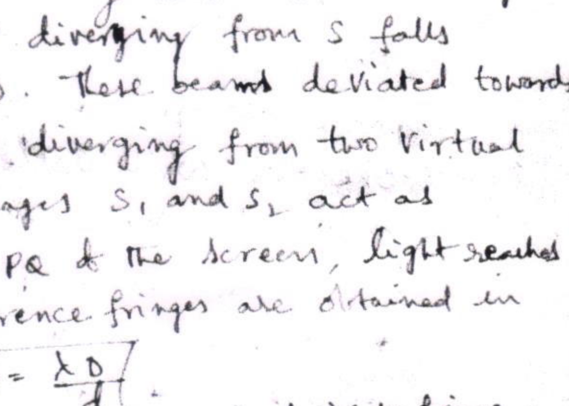


Fig (2) Thin film reflections

Reflected system:- Draw EP perpendicular to BC.

Then path difference between BC & EF = $\mu(BD+DE) - BP$.

Now $BD = DE = t/\cos(r)$ and $BQ = QE = t \cdot \tan(r)$

$$\therefore \quad \mu(BD+DE) = \frac{2\mu}{\cos r}$$

Also $BP = BE \sin i = (BQ+QE)\sin i$

$$= 2t \tan r \cdot \sin i$$

$$= 2t \frac{\sin r}{\cos r} \cdot [\mu \sin(r)]$$

$$\therefore \text{ path difference } = \frac{2\mu t}{\cos r} - \frac{2\mu t \sin^2 r}{\cos r} \quad \left(\because \mu = \frac{\sin(i)}{\sin(r)}\right)$$

$$= \frac{2\mu t}{\cos r}(1-\sin^2 r)$$

$$\therefore \boxed{\text{path difference} = 2\mu t \cos r}$$

Now, in the case of transverse waves, an additional phase change of π or path difference of $\lambda/2$ is created when reflection takes place at the surface of a Denser medium. (not with rarer medium)

The ray 'BC' suffers reflection on the surface of denser medium; hence it suffers a path difference of $\lambda/2$.

The ray 'BD' suffers reflection on the surface of rarer medium; hence it suffers no additional path difference.

Hence net path difference $= 2\mu t \cos r + \lambda/2$

(i) For a Thin film $t \approx 0$; $2\mu t \cos r = 0$
When the film is extremely thin: ie., $t = 0$;

The net path difference $= \lambda/2$

Thus the two rays produce Destructive interference

The film appears Dark

(ii) Film not Thin :-

The film appears bright if the path difference

$$2\mu t \cos r + \lambda/2 = n\lambda$$

$$\boxed{2\mu t \cos r = n\lambda - \lambda/2 = (2n-1)\lambda/2}$$

where $n = 1, 2, 3, \ldots$

(iii) The film appears dark if the path difference

$$2\mu t \cos r + \lambda/2 = (2n+1)\lambda/2$$

$$\boxed{2\mu t \cos r = n\lambda}$$

where $n = 0, 1, 2, 3, \ldots$

Vertically ...

Transmitted System :-

Path difference between the two transmitted rays DK and GH.

$$\mu(DE+EG) - DL = 2\mu t \cos r$$

At E the reflection is on the surface of a rarer medium hence no additional path difference.

(iv) When the film is extreemly thin ($t = 0$)
∴ The net path difference $= 0$ ∴ The film appears bright.

(v) When the film is not thin.
The film appears bright if the path difference

$$2\mu t \cos r = n\lambda \quad [n = 1, 2, 3 \ldots]$$

(vi) The film appears dark if the path difference

$$2\mu t \cos r = (2n-1)\lambda/2, \quad \text{where } n = 1, 2, 3, \ldots$$

∴ If a bright fringe is produced in the reflected system, a dark fringe will be produced in the transmitted system for the same path difference. Thus the reflected and transmitted systems are complementary.

Production of colours in thin films :-

With monochromatic light, the fringes are only bright and dark. However, with white light, the fringes are coloured : It is because path difference depends on

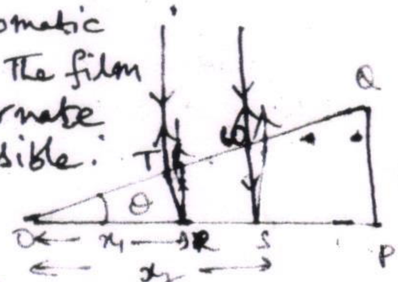
(i) If 't' and 'r' are constant, the path difference proportional to μ with white light the colours will appear in the order Violet, Blue etc, as λ increases.

(ii) If 'i' changes, 'r' also changes ; hence p-d changes. If we view the film in various directions, different colours will be seen.

(iii) When 't' varies, the film passes through various colours for the same 'i'

Wedge shaped film :--

Consider a wedge shaped film of refractive index μ enclosed by two plane surfaces OP and OQ inclined at an angle θ. The thickness of the film increases from O to P.



A parallel beam of monochromatic light falls on the film normally; the film is viewed in reflected light; alternate dark and bright bands will be visible. At R, the thickness is 't', R is at a distance 'x_1' from O.

The path difference between the rays: one reflected at 'T' and the other reflected at 'R' $= 2\mu t + \lambda/2$

where $\lambda/2$ dues to reflection at 'R' takes place at the surface of a denser medium (phase change of π rad or $\lambda/2$)

'R' will be appear dark band, if $2\mu t + \lambda/2 = (2n+1)\lambda/2$;

or $\boxed{2\mu t = 2n}$

R will be appear bright band, if $2\mu t + \lambda/2 = n\lambda$

or $\boxed{2\mu t = (2n-1)\lambda/2}$

but $\frac{t}{x_1} = \theta \quad \therefore t = x_1\theta$.

or $2\mu x_1 \theta = n\lambda \longrightarrow (1)$.

Similarly, for the $(n+1)^{th}$ dark band which is formed at 'S' at a distance x_2 from 'A', we have

$$2\mu x_2 \theta = (n+1)\lambda \longrightarrow (2)$$

subtracting eqn (1) from (2)

$$2\mu\theta(x_2 - x_1) = \lambda$$

fringe width $\beta = (x_2 - x_1) = \lambda/2\mu\theta$.

Similarly if we consider two consecutive bright fringes, the fringe width 'β' will be same.

Wedge shaped film is formed by inserting a thin paper or thin wire between the two plane glass plates.

For air film $\mu = 1$ & $\theta = t/x$

where 't' is the thickness of the thin paper; 'x' its distance from the edge where the two plates touch each other.

Experiment to measure the diameter of a thin wire :-

An air wedge is formed by inserting the thin wire between the two glass plates. Monochromatic light is reflected vertically downwards on to the wedge by the inclined glass plate

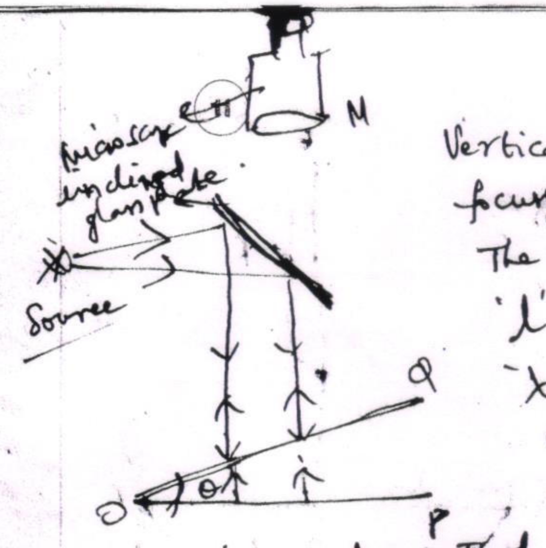


Fig wedge method

A travelling microscope M with its axis vertical is placed above G. The microscope is focussed to get clear dark and bright fringes. The fringe width β is measured. The length 'l' of the wedge also is measured. Knowing 'λ' the diameter (d) of the wire is calculated using the formula

$$\text{diameter of the wire } 'd' = \frac{\lambda L}{2\beta}.$$

Newton's Rings :— A plano convex lens of large radius of
curvature is placed with its convex surface in contact with a
plane glass plate, a plano concave shaped thin air film is formed
between the glass plate and the convex lens. The thickness of the
air film is zero at the point of contact O and gradually increases
from the point contact outwards as shown in fig (1).

When the monochromatic light is allowed to fall normally
on this film, a system of alternate bright and dark fringes of
concentric rings are formed in the air film. These circular
interference fringes are called Newton's rings.

Fig (1) Newton's rings.

The thickness of air film remains constant along a circle with its centre at 'O'. Hence, the fringes are in the form of concentric circles.

Newton rings are formed as a result of interference between the light waves reflected from the upper and lower surface of the air film due to amplitude division of the interfering rays corresponding to an incident ray.

Expression for radii of the rings and wavelength of light used.

Let 'R' is the radius of curvature of the lens and C be the
centre of the curvature as shown in fig (2). In fig (2).
AOB – vertical section of the lens, MON – plane glass plate
O – point of contact and AM = BN = t, thickness
of the air film at M or N. All points having the same

Thickness as AM lie along a circle with 'O' as the centre.

Path difference between the rays one reflected from A and the other from M is $2\mu t \cos r$.

where μ for air is 1, for normal incidence $r = 0$

$\therefore$ path difference $= 2t$.

At 'M' or 'N' the ray suffers reflection on the surface of the denser medium

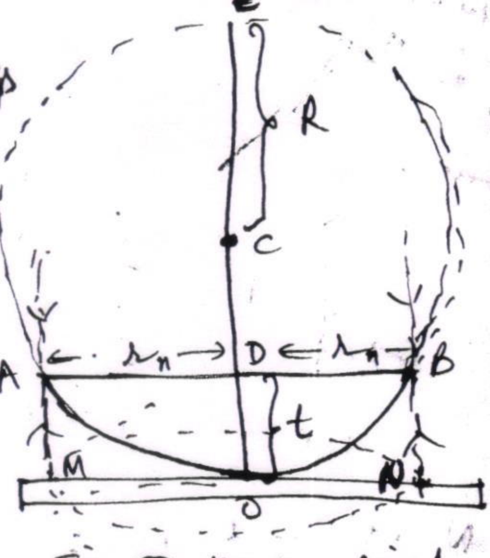


Fig ② Newton rings.

$\therefore$ The total path difference $= 2t + \lambda/2$

The condition for the bright fringe is for the n^{th} ring of diameter MN.

$$2t + \lambda/2 = n\lambda$$

$$\boxed{2t = (2n-1)\lambda/2} \rightarrow \text{for bright fringe}$$

The condition for the dark fringe is

$$2t + \lambda/2 = (2n+1)\lambda/2$$

$$\boxed{2t = n\lambda} \longrightarrow \text{for dark fringe}$$

From the Geometry of the circle—

$$AD \times DB = OD \times DE$$

$$r_n^2 = t(2R - t) = 2Rt - t^2$$

Since OD thickness of the film (t) is small, t^2 is very small, it is neglected when compared to $2Rt$

$$\therefore r_n^2 = 2Rt.$$

$\therefore$ Radius of the n^{th} ring or r_n $\boxed{r_n = \sqrt{2Rt}}$

$\therefore$ n^{th} bright ring diameter d_n.

$$\frac{d_n^2}{4} = 2Rt, \quad \text{or} \quad \frac{d_n^2}{4R} = 2t = (2n-1)\lambda$$

$$\therefore \frac{d_n^2}{4R} = (2n-1)\lambda/2 \longrightarrow ①.$$

for m^{th} bright ring: $\dfrac{d_m^2}{4R} = (2m-1)\lambda/2 \longrightarrow ②$.

from equ (1) and (2)

$$\frac{d_n^2 - d_m^2}{4R} = (n-m)\lambda$$

Wave length of light used

$$\boxed{\lambda = \frac{d_n^2 - d_m^2}{4R(n-m)}}$$

The result is same for the n^{th} and m^{th} dark rings.
Radius of curvature 'R' is measured with Spherometer hence λ can be determined.

(i) In case of reflection of light at lens the central spot is dark.

(ii) In case of transmition of light at lens, the central spot is bright.

(iii) Only with lens having large R the clear rings are formed, so that the measurements can be made easily.

(iv) large decreases 't' at any point so we can neglect t^2

(v) angle of wedge between plane glass plate and the lens is neglected.

(vi) When transparent material of (μ) instead of air film enclosed. The path difference $= 2\mu t$ (instead of $2t$)

$$\therefore \mu = \frac{(d_n^2 - d_m^2)}{(d_n'^2 - d_m'^2)}$$

$$\boxed{\therefore \lambda = \mu \frac{(d_n'^2 - d_m'^2)}{4R(n-m)}}$$

$\lambda/2$

(4)

If a white screen is placed in the region beyond the slits, a pattern of bright and dark fringes are seen on the screen. When a crest falls on a crest or a trough falls on a trough produces a constructive interference and hence gives bright fringe. When a crest falls on a trough, produces a destructive interference which gives a dark fringe. Thus, the bright and dark fringes occur alternately at equal distances as shown in fig ①

Theory of Young's double slit experiment :-

S is a source of monochromatic light of wavelength λ. S_1 and S_2 are two narrow slits equidistant from S. The ray diagram is shown in fig ②.

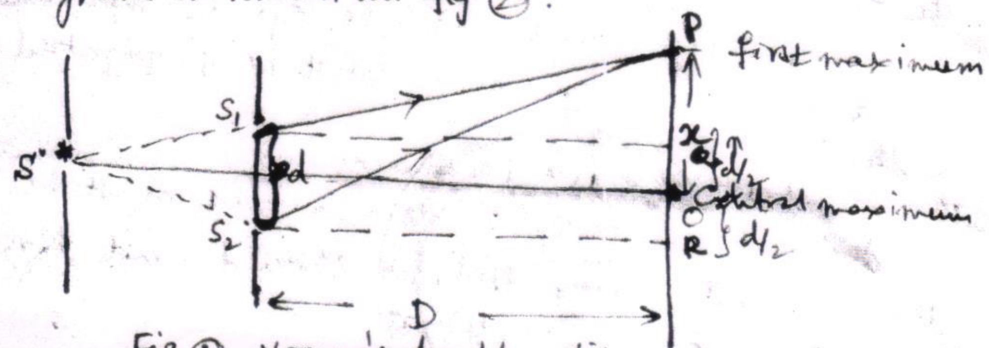


Fig ② Young's double slit experiment ray diagram.

The path difference be $S_1O - S_2O = 0$.

The path difference between the two waves $= S_2P - S_1P$.

$PQ = x - d/2$ and $PR = x + d/2$

$PR = x + d/2$

$(S_2P)^2 = D^2 + (x + d/2)^2$ and $(S_1P)^2 = D^2 + (x - d/2)^2$

$(S_2P)^2 - (S_1P)^2 = [D^2 + (x+d/2)^2] - [D^2 + (x - d/2)^2] = \frac{4xd}{2}$

$\therefore S_2P - S_1P = \frac{2xd}{(S_2P + S_1P)}$ $\quad (\because (S_2P)^2 - (S_1P)^2 = (S_2P + S_1P)(S_2P - S_1P))$

$\therefore$ path difference $S_2P - S_1P = \frac{2xd}{2D}$ $\quad (\because S_2P \approx S_1P = D)$

$$\boxed{S_2P - S_1P = xd/D} \rightarrow (1).$$

$\therefore$ phase difference $$\boxed{\phi = \frac{2\pi}{\lambda}\left(\frac{xd}{D}\right)} \rightarrow (2).$$

Position of bright fringes :— If the path-difference is $n\lambda$, the point p is bright. Thus for n^{th} bright fringe,

$$\frac{x_n d}{D} = n\lambda \quad \text{where } n = 0, 1, 2, 3, \ldots$$

or $$\boxed{x_n = \frac{D\lambda}{d} \cdot n} \longrightarrow (I).$$

The central bright fringe ($n=0$) is called the zeroth-order maximum.

The bright fringes with $n = 1, 2, 3, \ldots$ are called the first, second, third etc., order fringes and their positions.

$$x_1 = \frac{D\lambda}{d}, \quad x_2 = \frac{2D\lambda}{d}, \quad x_3 = \frac{3D\lambda}{d} \ldots x_n = \frac{nD\lambda}{d}.$$

$$x_{n+1} = \frac{D}{d}(n+1)\lambda$$

$$\therefore \text{Fringe width}(\beta) = x_n - x_{n-1} = \frac{nD\lambda}{d} - \frac{(n-1)D\lambda}{d} = \frac{\lambda D}{d}.$$

$$\therefore \boxed{\beta = \frac{\lambda D}{d}}$$

Position of dark fringes :— If the path-difference is odd multiple of $\lambda/2$, the p is dark. Thus for n^{th} dark fringe

$$\frac{x_n d}{D} = (2n-1)\lambda/2 \quad \text{where } n = 1, 2, 3, \ldots$$

The dark fringes with $n = 1, 2, 3 \ldots$ are called the first second, third etc., order fringes and their positions.

$$x_1 = \frac{\lambda D}{2d}, \quad x_2 = \frac{3\lambda D}{2d}, \quad x_3 = \frac{5\lambda D}{2d} \ldots x_n = \frac{(2n-1)\lambda D}{2d}$$

$$\therefore x_n = \frac{(2n-1)\lambda D}{2d} \quad \text{and} \quad x_{n-1} = \frac{2(n-1)-1\,\lambda D}{2d} = \frac{(2n-3)\lambda D}{2d}$$

$$\therefore \text{Fringe width}(\beta) = x_n - x_{n-1} = \frac{[(2n-1) - (2n-3)]\lambda D}{2d} = \frac{2\lambda D}{2d}$$

$$\therefore \boxed{\beta = \frac{\lambda D}{d}}$$

Since D, d, λ are constants, fringe width β is constant. The fringe width β is same for bright fringes and dark fringes.

The fringe width 'β' is directly proportional to 'D' and 'λ' and inversely proportional to 'd'. Thus for interference 'd' should be small as possible.

If the source is white light (Mercury lamp) the central bright fringe is white and others are coloured (VIBGYOR)

Michelson's Interferometer :-

principle :- Interferometers are constructed on the principle of interference of light by division of amplitude.

Construction :- Michelson interferometer consists of two plane glass plates G_1 & G_2 parallel to each other, identical in every respect. G_1 is partly silvered so that light coming from 'S' is partly reflected and partly transmitted. Both the glass plates are inclined at an angle of 45° to the mirrors M_1 and M_2 which are mutually perpendicular to each other. M_1 is mounted on a carriage and can be moved parallel to itself. It's displacement can be read upto 10^{-5} of a centimeter. Both the mirrors are provided with levelling screws by which they can be made perpendicular to the direction of light beams. Light reflected from both the mirrors interfere in the field of view of the telescope 'T'.

working :- Light from the source S is rendered parallel by a lens L and falls on the glass plate G_1 at an angle of 45°. It is partly reflected along G_1M_1 and partly transmitted along G_1M_2. These two rays are reflected back by the two mirrors along the same path and received in the telescope along G_1T. The ray G_1M_1 traverses the glass plate A twice; but G_1M_2 traverses G_1 only once. So to compensate, an identical glass plate G_2 is introduced.

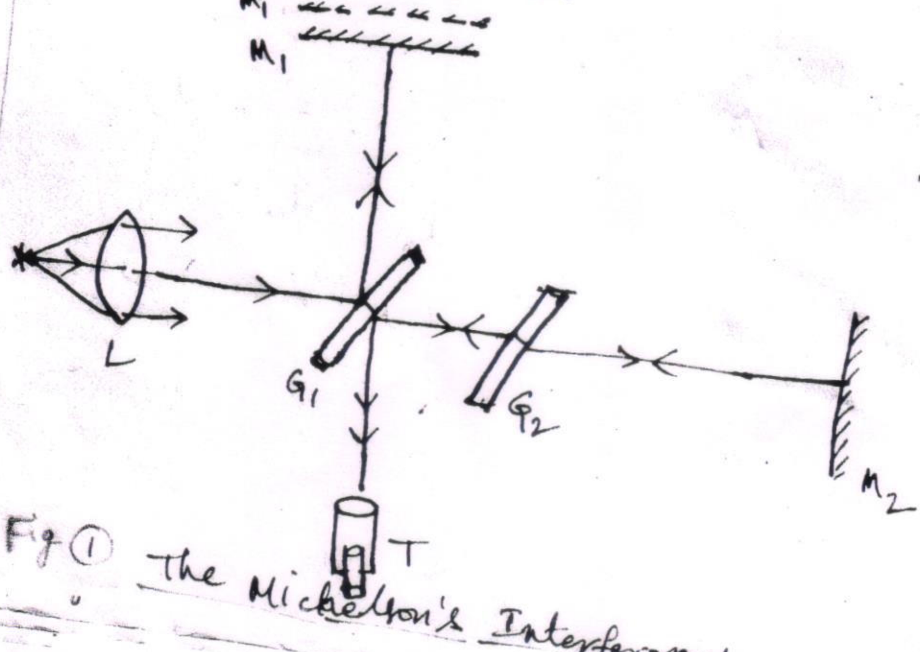


Fig ① The Michelson's Interferometer.

(15) with the help of the levelling screws, the mirrors are made optically perpendicular to the light beams. A slight motion of M_1 parallel to itself produces circular fringes, given by a path difference of $2d\cos\alpha + \lambda/2 \longrightarrow (1)$. Where α is the angle of the eye or the telescope through which it making an angle with the normal to the mirrors. The ray M_2G_1 suffers reflection at the surface of a denser medium at G_1, an additional path difference of $\lambda/2$ is introduced between the two rays. Bright fringes are produced if $\boxed{2d\cos\alpha + \lambda/2 = n\lambda}$ where $n = 0, 1, 2, 3, \cdots$, and dark fringes are produced if $\boxed{2d\cos\alpha + \lambda/2 = (2n+1)\lambda/2}$

By tilting the mirror M_2 slightly, the circular fringes are made practically straight. (The air path between the two mirrors becomes wedge shaped). With white light, a few coloured fringes with a central dark fringe is observed.

<u>To find the wavelength of monochromatic light</u>:- With circular fringe formation, the cross wire is coincided on a particular bright fringe, when M_1 is moved through $\lambda/2$, the path difference changes by 'λ' and the next bright fringe comes at the cross-wire. When 'n' fringes move as M_1 is moved through 'l' distance, then $n.\lambda/2 = l$ So $\boxed{\lambda = 2l/n}$

<u>Determination of thickness of transparent plate of refractive index</u>:-

When the plate is introduced in one of the light paths, the optical path in this is increased as light travels slowly through the medium of μ; now the path in the medium is 'μt' instead of 't' in air: So, the increase in the optical path $= (\mu - 1)t$.

First, the cross-wire is coincided with the dark fringe of zero path (central). Now the introduction of glass plate the fringes move; Now M_2 is moved till again the dark fringe of zero path is again at the cross wires. If M_2 is moved over a distance 'l' it gives the increase in path $(\mu - 1)t = l$. If 'μ' the refractive index is known, 't' is determined.

DIFFRACTION

The phenomenon of bending of light at sharp corners and spreading of light into the geometrical shadow of an object is called diffraction.

There are two types of diffraction

(i) Fresnel's diffraction and (ii) Fraunhofer's diffraction.

(i) Fresnel's diffraction :- In this, the source and the screen are placed at finite distances from the aperture having sharp edges. No lenses are used for making the rays convergent. The incident wavefront is spherical or cylindrical.

(ii) Fraunhofer's diffraction :- In this case, the source and the screen are at infinite distances from the aperture. The lenses are used between the source and the screen. The incident wavefront is plane.

Differences between interference and diffraction :-

Interference	Diffraction
(i) The modification of light intensity by the superposition of two or more waves from coherent sources is called the interference.	(i) The bending of light at sharp edges of the obstacle is called the diffraction.
(ii) Interference is the result of interaction of light coming from two different wavefronts originating from the same source.	(ii) Diffraction is the result of interaction of light coming from different parts of the same wavefront.
(iii) fringes are of the same width	(iii) fringes are not of the same width.
(iv) points of minimum intensity are perfectly dark.	(iv) points of minimum intensity are not perfectly dark.
(v) All bright bands are of uniform intensity	(v) bright bands intensity decreases with order
(vi) There is no diffraction phenomenon in the interference fringes.	(vi) In the diffraction the fringes are formed due to interference

6

Fraunhofer Diffraction at a Single Slit :-

A plane wavefront is incident on the slit 'AB' $\frac{=}{5}$ a. Light in passing through the slit suffers diffraction. The diffracted light is focused by a convex lens on a screen placed in the focal plane of the lens. The diffraction pattern obtained on the screen consists of a central bright band, having alternate dark and weak bright bands of decreasing intensity on both sides.

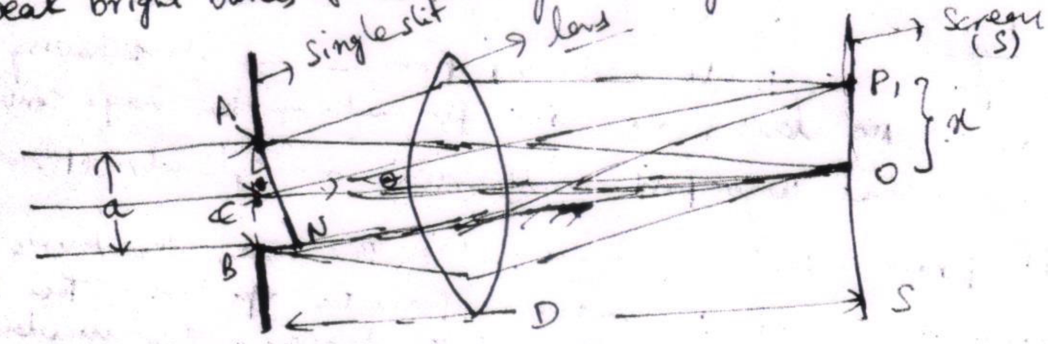


Fig ① Diffraction by Single Slit.

The secondary wavelets travelling parallel to 'CO' converges to the central point 'O' forming a bright central image at 'O', since the wavelets in portions 'CA' & 'CB' travel the same distance in reaching 'O'. The lens forms a real image on the screen (S) (central bright image).

Now, the secondary wavelets making an angle 'θ' at 'C' converges to a point 'P'; thus P is bright or dark depending on the path difference. A perpendicular 'AN' is drawn to the diffracted rays. The path difference from A and B is $BN = AB \sin\theta = a \sin\theta$.

If $BN = \lambda$, the path difference between the waves from A and C is '$\lambda/2$'. Similarly for the lower half ~~CB~~ C and B is $\lambda/2$, gives the destructive interference. Hence for the first minima $BN = a\sin\theta = \lambda$

or $\sin\theta = \lambda/a$ or $\theta = \lambda/a$ ($\because$ θ is very small)

Hence the first minimum on either side of 'O' will occur in a direction given by $\theta = \lambda/a$

Thus, the successive minima are obtained for a path difference of 2λ, 3λ and 4λ etc.,

Thus for the n^{th} diffracted minima, the general formula is

$$BN = a\sin(\theta_n) = n\lambda \quad \text{or} \quad \sin\theta_n = \frac{n\lambda}{a}$$

Besides the central maximum at 'O' there are secondary maxima which lie in between the minima on either side of the 'O'.

For secondary maxima, $BC = a\sin\theta = (2n+1)\lambda/2$ for odd multiples of wavelength (λ) where n = 1, 2, 3 etc., The intensity of these secondary maxima is much less and ~~falls~~ the intensity decreases rapidly at ~~more~~ second order ..., as shown in fig ②.

'Y' dues to both the slits diffracted at an angle θ ;

$$y = y_1 + y_2 = A \sin \omega t + A \sin(\omega t + 2\beta)$$
$$= 2A \cos\beta \cdot \sin(\omega t + \beta) \ ;$$

Here, amplitude $= 2A\cos\beta$; Hence intensity $= 4A^2\cos^2(\beta)$;

Thus the resultant intensity in the Diffraction pattern is due to (i) diffraction pattern due to a single slit ($4A^2$) (ii) interference pattern due to wavelets from the corresponding points of the two slits.

The intensity diagram is shown in fig ② when $\beta = 0$ we get the intensity of the central maximum; the full line curve represents the intensity distribution due to interference from both the slits; the diffraction maxima and minima is shown by the dotted curve. Within each diffraction maxima is seen equally spaced interference maxima and minima the spacing between interference maxima and minima depends on the values of 'a' and 'b'. The intensity pattern shown below is for $2a = b$.

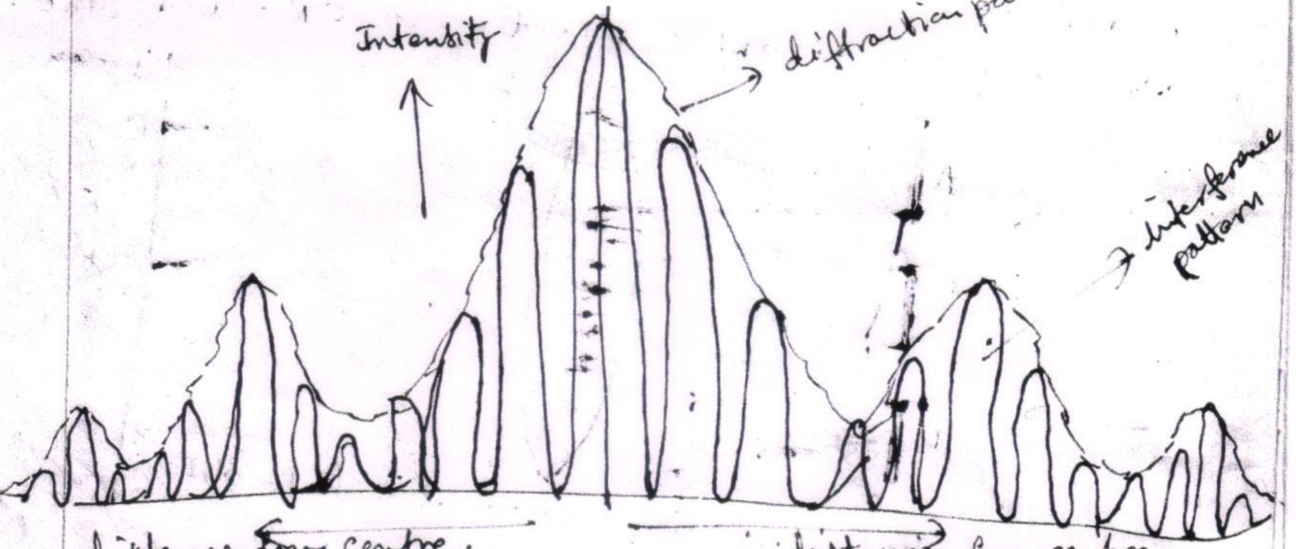


Fig ② The intensity distribution pattern due to a double slit

Second order (n = 2)

7

Let f be the focal length of the lens and let x be the distance of first minimum from O. Then the width of the central maximum is $2x$. If the lens is close to the slit then $\sin\theta = \lambda/a = \frac{x}{f}$ ($\because D = f$)

$\therefore x = \frac{f\lambda}{a}$

$\therefore$ width of the central maximum $= \boxed{2x = \frac{2f\lambda}{a}}$

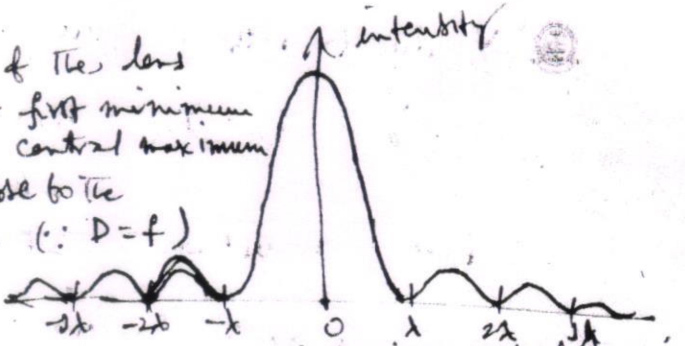


Fig ① intensity of diffraction pattern.

Hence the width of the central maximum is proportional to the wavelength of light λ.

With monochromatic source, on the either side of the central maximum, there are alternate bright and dark bands are formed.

Fraunhofer Diffraction at a double slit :-

Two parallel rectangular slits 'AB' and 'CD' of equal width 'a' separated by a opaque distance 'b' as shown in fig. ①. A plane wavefront is incident on the double slit from a monochromatic source. Suppose each slit diffracts the beam in a direction making an angle 'θ' with the direction of the incident beam. Then the path difference between A and C of the two slits $CN = (a+b)\sin\theta$.

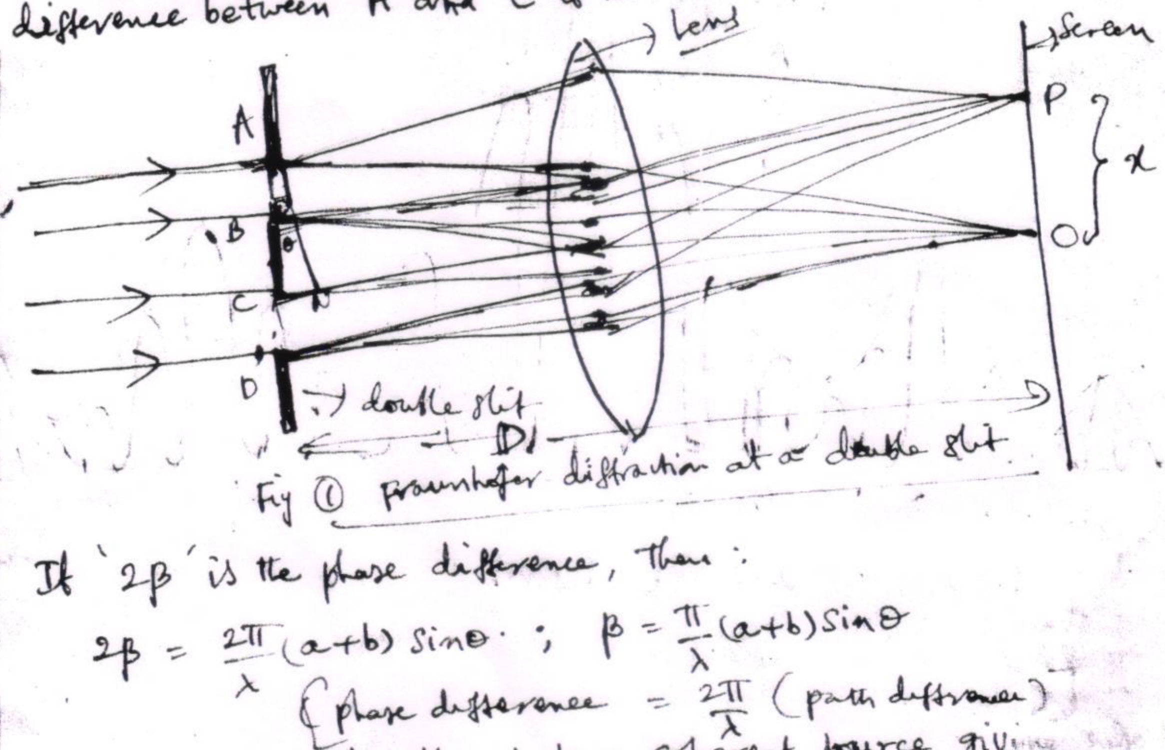


Fig ① Fraunhofer diffraction at a double slit

If '2β' is the phase difference, then :

$$2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta \; ; \quad \beta = \frac{\pi}{\lambda}(a+b)\sin\theta$$

$$\left\{ \text{phase difference} = \frac{2\pi}{\lambda}(\text{path difference}) \right.$$

If each slit is small and to a coherent source giving interference pattern on the screen. The displacement due to first slit $Y_1 = A\sin(\omega t)$. The displacement due to second slit $Y_2 = A\sin(\omega t + 2\beta)$. Hence, the resultant

9

Diffraction Grating :-

An arrangement consisting of a large number of parallel slits of equal width and separated from one another by equal opaque spaces is called a "diffraction grating".

It is constructed by ruling with a diamond point, a large number of equidistant parallel lines on a tranparent material like a glass plate: Usually, there are 15,000 lines per inch in a laboratory grating. The ruled lines act like opacities: light passes through the space between the lines called transparancies. Such a grating is called "transmission grating"

Let the width of the transparancy and opacity be 'a' and 'b' respectively. The distance (a+b) is called "Grating Element" or "Grating constant". The points separated by (a+b) are called "corresponding points".

Let a parallel beam of monochromatic light (plane wavefront) falls normally on the grating surface as shown in fig ①.

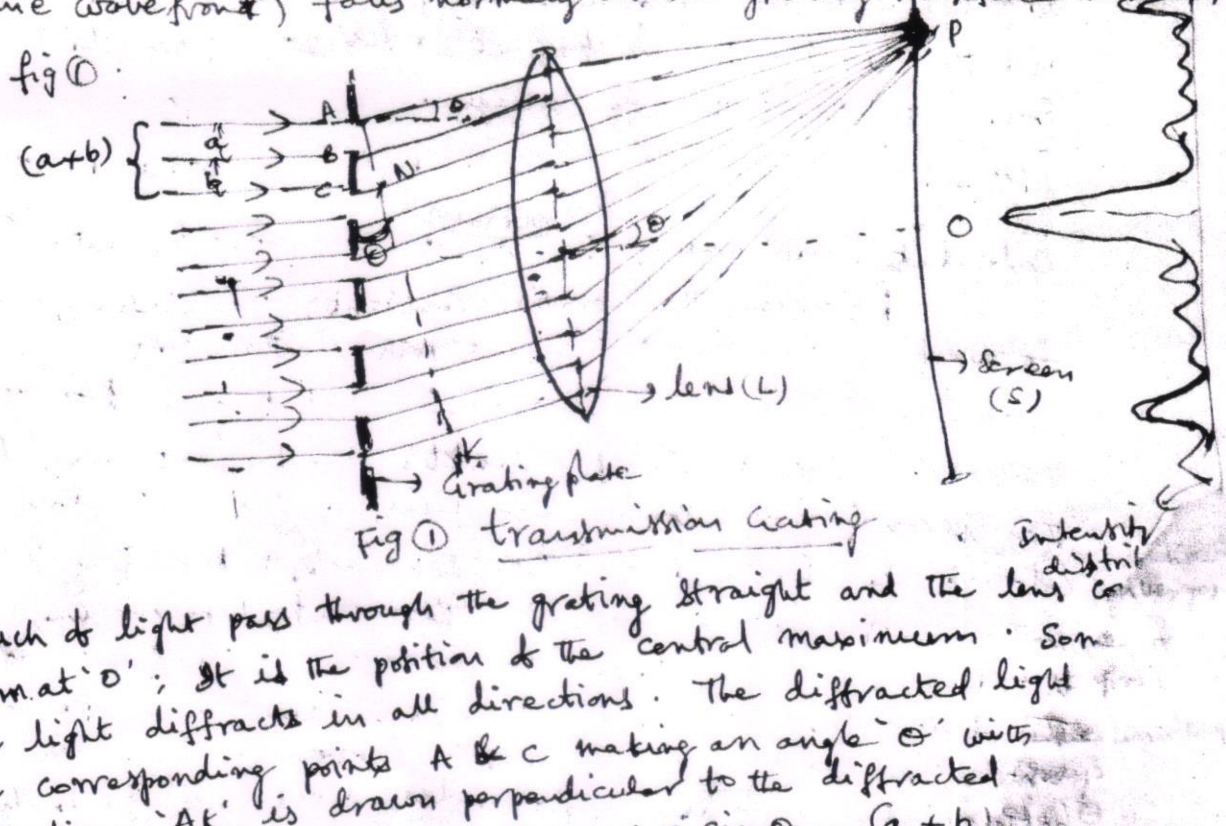


Fig ① transmission Grating

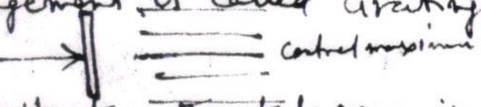
Much of light pass through the grating straight and the lens converge them at 'O'; It is the position of the central maximum. Some of the light diffracts in all directions. The diffracted light from the corresponding points A & C making an angle 'θ' with the... 'AK' is drawn perpendicular to the diffracted direction.

Then path difference $CN = AC \sin\theta = (a+b)\sin\theta$

Hence for maximum intensity $(a+b)\sin\theta = n\lambda$ ... 2, 3, --etc., . For $n=0$, the central maximum at 'O' and $n=1$ gives rise to first order maximum

Determination of the wavelength using Grating on a Spectrameter:- The initial adjustments of the Spectrameter are made. The levelling screws are adjusted such that the diffracted images both on the left and on the right side of the central image, with grating mounted. The left side and right side image should be are on the field of the telescope.

To set the grating for normal incidence of light, the telescope is turned to catch the direct image at the cross wires. Now, the telescope is turned exactly through 90° and fixed. Now, the spectrameter base is turned until the reflected image of the slit is caught on the vertical cross-wire. Now the ~~prism~~ Grating table is turned through 45° when the plane of the grating is perpendicular to the axis of the collimator. This arrangement is called Grating is at normal incidence.



To find the grating constant, the telescope is turned and the first order image readings are taken both on the left and the right. The difference gives "2θ" from which 'θ' is found for the first order image. Using the formula

$$\boxed{(a+b)\sin\theta = \lambda}$$

the grating element $(a+b)$ is calculated assuming the 'λ' of the monochromatic source.

Knowing the grating constant, the telescope is focused on the various lines both in the first order and the second order and the readings are taken in both the verniers. The telescope starts from the extreem left and the second order comes into field of view; the various colours are focussed and the readings are taken, then first order in the left side are covered. Then central bright image reading is taken. Then the telescope is moved to the right side and the first order is covered; then it is still moved further until the right side II order is covered. The difference between the left and right sides gives '2θ' for each colour and for each order. Then 'θ' for each colour is determined. Substituting the value of 'θ' in the formula $\boxed{(a+b)\sin\theta = n\lambda}$ we can determine the wavelength of the various colours of the mercury spectrum both in the first order ($n=1$) and in the second order ($n=2$).

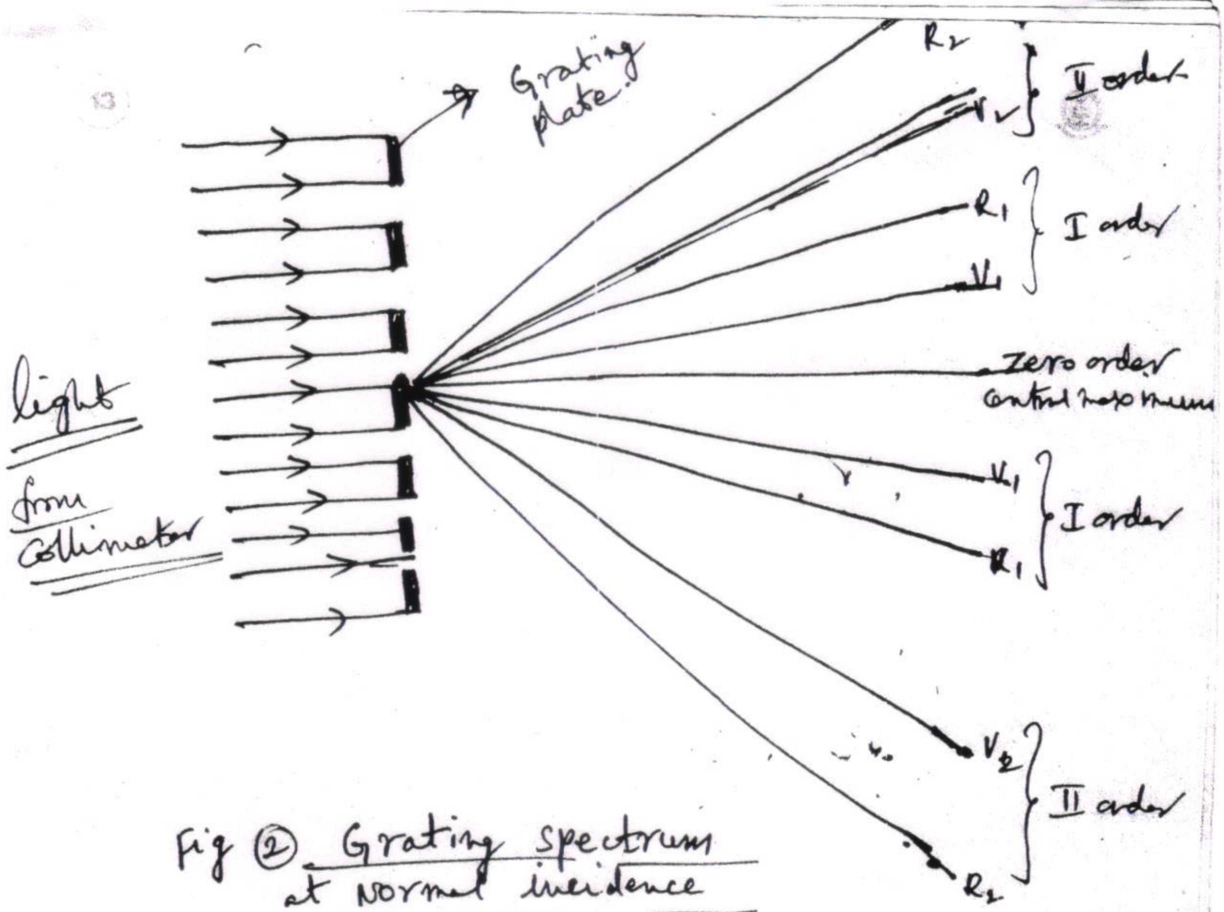


Fig ② Grating spectrum at normal incidence

Minimum deviation method :—

The telescope is turned and the central bright image of the slit is located. The telescope is turned further away and the first order image is located. The grating table is rotated in such a direction as to shift the image towards the axis of the collimator. This is continued until the image reaches its limiting position called minimum deviation. The telescope is turned using the fine motion screw until the point of intersection of the crosswires coincides with the image. The reading R_1 on the circular scale is noted.

Now the telescope is turned to get the direct image of the slit at the point of intersection of the crosswires. The reading R_2 on the circular scale is noted. Then the angle of minimum deviation of the first order image is $D = R_1 \sim R_2$. The wavelength λ of the light is calculated using the formula

$$\boxed{2(a+b)\sin(D/2) = n\lambda}$$

where $(a+b)$ is the grating element by assuming green line wavelength as 5461 Å. $(a+b)$ can be calculated. Therefore the number of lines/cm (N) calculated from the formula

$$\boxed{N = \frac{1}{(a+b)} = \frac{2\sin(D/2)}{n\lambda}}$$

Resolving power of Optical Instruments :-

The capacity of an instrument to show two close things separately is called "resolution". The ability of an optical instrument to produce distinctly separate spectral lines of light having two or more wavelengths very close to each other or to resolve the images of two nearby points is called its "resolving power".

Rayleigh's criterion for resolution :- Two spectral lines of equal intensities (two point sources) are just resolved by an optical instrument when the principal maximum of the diffraction pattern due to one falls on the first minimum of the diffraction pattern of the other.

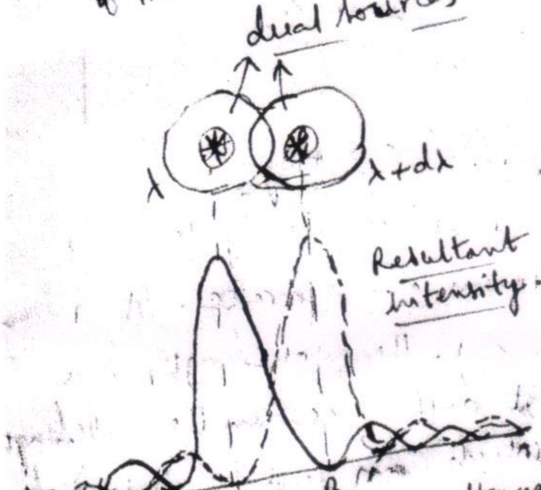


Fig ① Diffraction pattern of the dual sources

Limit of Resolution and Resolving powers of Microscope and Telescopes :-

When a beam of light passes through the objective of the telescope, the lens act like a circular aperture producing a diffraction pattern instead of a point image. If there are two point objects lying very close to each other, then the diffraction pattern produced by them will overlap.

According to Raleigh criterion, the images of the point objects are said to be separated if the central maxima of the one falls on the first secondary minima of the other as shown in fig ①. Thus the two sources (or spectral lines) are said to be resolved.

For rectangular apertures $\boxed{d\theta = \lambda/D}$

In a telescope, this smallest angular separation is given by $\boxed{d\theta = \frac{1.22\lambda}{d}}$ where d is the diameter of the objective lens. [for circular apertures Airy showed that $\boxed{d\theta = 1.22(\lambda/D)}$]

The resolving power = The reciprocal of the smallest angular separation

$$R.P = \boxed{\frac{1}{d\theta} = \frac{d}{1.22\lambda}}$$

Thus resolving power depends on the diameter of the objective & on λ.

4

Resolving power of a Microscope :- The minimum distance by which two points in the object are separated from each other so that their images as produced by the microscope are just seen as separate is called the limit of resolution of the Microscope. The reciprocal of the limit of resolution is called resolving power of the microscope.

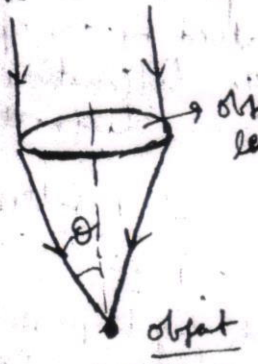


Fig ② Microscope

When θ is the angle of incidence of light on the object

$$2d \sin\theta = 1.22\lambda$$

Limit of resolution $\boxed{d = \dfrac{1.22\lambda}{2\sin\theta}}$

Resolving power of a plane Diffraction Grating :-

The R.P of a grating is defined as its ability to show two neighbouring lines in a spectrum as separate. It is measured by the ratio $\lambda/d\lambda$, where λ is the wavelength of a spectral line and $d\lambda$ is the least difference in the wavelength of two neighbouring spectral lines which can just be resolved.

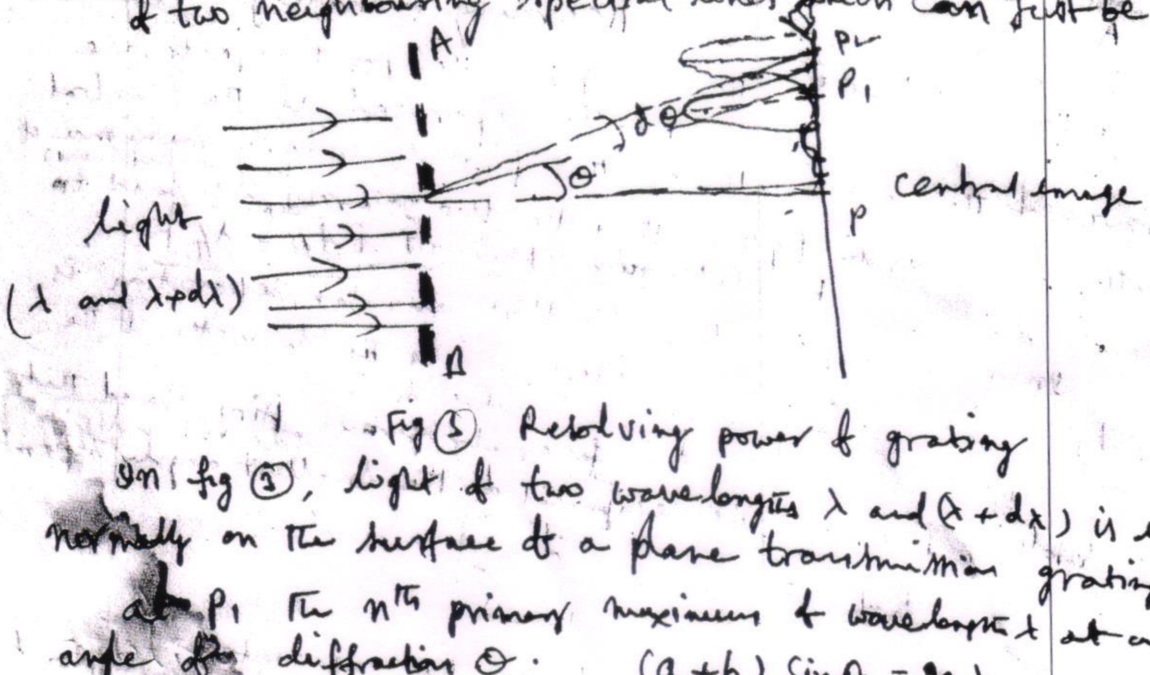


Fig ③ Resolving power of grating

In fig ③, light of two wavelengths λ and $(\lambda + d\lambda)$ is incident normally on the surface of a plane transmission grating AB.

at P_1 the n^{th} primary maximum of wavelength λ at an angle of diffraction θ.

$$(a+b)\sin\theta = n\lambda \longrightarrow ①$$

at P_2 wavelength $(\lambda + d\lambda)$ at an angle of diffraction $(\theta + d\theta)$

Thus

$$(a+b)\sin(\theta + d\theta) = n(\lambda + d\lambda) \longrightarrow ②$$

5

According to Rayleigh, the two spectral lines will appear just resolved if the principle maximum due to $(\lambda + d\lambda)$ falls on the first minimum of λ or vice versa. Thus the two lines will appear just resolved if the angle of diffraction $(\theta + d\theta)$ also corresponds to the direction of first secondary minimum after the n^{th} primary maximum at P, corresponding to wavelength λ. This is possible if the extra path difference introduced is λ/N. Here N is the total number of lines in the grating.

$$\therefore (a+b)\sin(\theta + d\theta) = n\lambda + \lambda/N$$

Equating the right hand side of eqn (2) & (1).

$$n(\lambda + d\lambda) = n\lambda + \lambda/N \quad \text{or} \quad n\,d\lambda = \lambda/N$$

∴ Resolving power ~~of grating (R.P)~~

$$\therefore R.P \quad \boxed{\lambda/d\lambda = nN}$$

The R.P increases with

(i) The order of the spectrum and

(ii) The total number of lines N on the grating

The R.P is independent of the grating element $(a+b)$.

The differences between Grating and prism spectra

Grating spectra	Prism spectra
(1) A number of spectra I order, II order etc., on both sides of central maximum.	(1) Only one spectrum.
(2) Deviation is proportional to wavelengths of light; inversely proportional to grating element; independent of the grating material.	(2) Deviation is more for violet than red, depends on refractive index of the material of the prism.
(3) Dispersive power (angular separation) between colours is practically same for all colours; for second order, it is double for first order. This increases as grating element $(a+b)$ decreases.	(3) Dispersive power is more for shorter wavelengths (violet) than for longer wavelength (red). Thus violet portion is more drawn out and less intense.
(4) Spectrum produced are similar.	(4) Neither similar nor regular
(5) Resolving power is very large ~~(N/n)~~ depends on N and n. Independent of grating element $(a+b)$ and ~~wavelength~~ wavelength λ.	(5) The resolving power very small, depends on length of the base, material of the prism & wavelength λ.

Lloyd's Single Mirror :— The Lloyd single mirror ~~to~~ produce, two coherent sources is shown in fig ①.

It consists of a narrow slit 'S' illuminated by the light source. The light waves from 'S' falls ~~at~~ on the polished surface AB as shown in fig ①. The mirror AB polished black at the bottom surface.

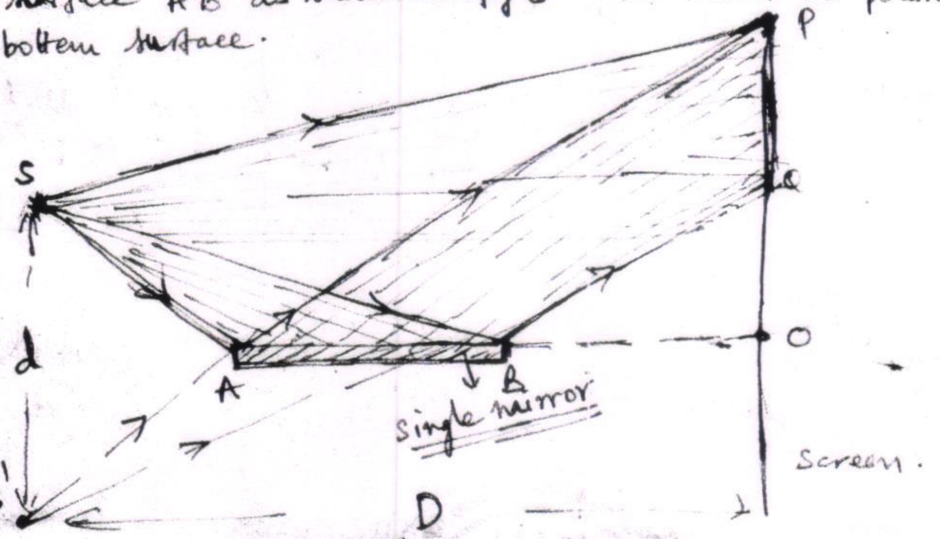


Fig ① Lloyd Single mirror

The interference takes place between the waves from 'S' and the waves reflected from AB. The reflected rays appear to come from the Virtual image S'. Thus S and S' are two coherent sources.

The interference pattern is observed on the screen in the region 'PQ'. The fringe system cannot extend below 'O' as no reflected wave can reach there.

When the screen at B, the central fringe is dark. Since due to reflection at B introduces an additional phase π or 180° or a path difference $\lambda/2$. Thus the condition for bright and dark fringes are reversed.

So, a phase difference of π, 3π, 5π or a path difference $\lambda/2$, $3\lambda/2$, $5\lambda/2$ --- produces bright fringes.

A phase difference of 0, 2π, 4π --- or a path difference 0, λ, 2λ, --- produces dark fringes.

The fringe width $\boxed{\beta = \frac{\lambda \cdot D}{d}}$

With mercury spectrum, the maxima of all colours superimposed and become white. Similarly at the minima of all colours superimposed and become dark. Thus white and black fringes are formed